Electronic transport through ferromagnetic and superconducting junctions with spin-filter tunneling barriers.

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We present a theoretical study of the quasiparticle and subgap conductance of generic $X/I_{sf}/S_M$ junctions with a spin-filter barrier I_{sf}, where X is either a normal N or a ferromagnetic metal F and S_M is a superconductor with a built-in exchange field. Our study is based on the tunneling Hamiltonian and the Green's function technique. First, we focus on the quasiparticle transport, both above and below the superconducting critical temperature. We obtain a general expression for the tunneling conductance which is valid for arbitrary values of the exchange field and arbitrary magnetization directions in the electrodes and in the spin-filter barrier. In the second part we consider the subgap conductance of a $N/I_{sf}/S$ junction, where S is a conventional superconductor. In order to account for the spin-filter effect at interfaces, we heuristically derive boundary conditions for the quasiclassical Green's functions. With the help of these boundary conditions we show that the proximity effect and the subgap conductance are suppressed by spin-filtering in a $N/I_{sf}/S$ junction. Our work provides useful tools for the study of spin-polarized transport in hybrid structures both in the normal and in the superconducting state.

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I. INTRODUCTION

Over the last decade, there has been a growing interest in studying superconductor/ferromagnet (S/F) hybrid structures. On the one hand, this interest is due to the progress in technology that allows a controllable fabrication of nanohybrid systems using a wide range of superconducting and magnetic materials. On the other hand, this interest is due to the discovery of new and interesting fundamental phenomena, as for example the so-called $\pi-$state in $S/F/S$ Josephson junctions[1–9], and more recently the long-range proximity effect mediated by odd-frequency triplet superconducting correlations in S/F structures[10–17] (for an overview see Refs.[18–22]).

The triplet superconducting correlations can carry spin-polarized supercurrents, i.e. currents without dissipation, that can be exploited in several ways in spintronics devices[20]. In this context, the use of tunnel barriers with spin-dependent transmission, the so-called spin filters, may be desirable for the creation of such spin supercurrents. Spin filters are tunnel barriers with spin-dependent barrier height. They have been used for decades to generate polarized currents in spintronic circuits[23,24].

In spite of numerous works devoted to the theoretical study of S/F structures, the study of the spin-filter effect in connection with the transport properties of S/F structures still remains open. For example, in Ref.[32] the transport properties of an S/F junction were calculated by using the Blonder-Tinkham-Klapwijk formalism[33]. This analysis was extended in several other works[27–31,34,35] for S/F and S/F/S junctions in the ballistic and diffusive limit by taking into account spin-active interfaces between the F and S layers.[25,26] In particular, the results for the diffusive limit presented in those works have been obtained by using the boundary conditions for the quasiclassical Green's functions derived in Ref.[29]. However, as we will show in section IV, these boundary conditions (BCs) cannot describe the spin-filter effects and hence, none of the above mentioned works addressed the question of how the spin filtering affects the proximity effect in S/F structures. Only recently, we have analyzed[36] the effect of spin filtering on the Josephson current through a $S/F-I_{sf}-F/S$ junction. It was shown that even in the case of a highly spin-polarizing barrier, a Josephson junction can flow provided the magnetizations of the F layers are non collinear. The results of Ref.[36] have been obtained from a model that combines the tunneling Hamiltonian and the quasiclassical Green's

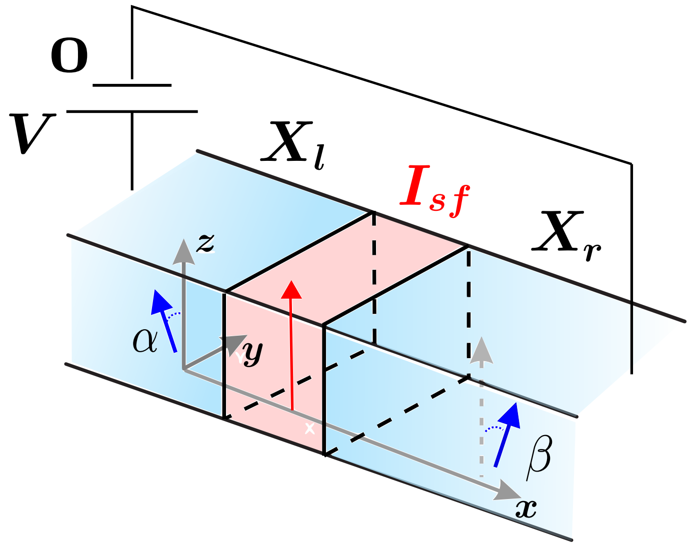

FIG. 1: A generic tunnel junction $X_l/I_{sf}/X_r$. The left and right electrodes, X_l and X_r, are either a ferromagnet F, normal metal N, or superconductor S_M with a built-in exchange field h. The layer I_{sf} is a spin-filter barrier. α and β are the orientations of the exchange fields in the left and right electrodes.

functions, and they provide a plausible explanation for a recent experiment on spin-filter Josephson junctions[37]. Note however, that the model used in Ref.[36] assumes exchange fields to be smaller than the Fermi energy and therefore cannot be straightforwardly generalized for the case of strong ferromagnets.

In the current paper, we present a general theory for the conductance through different hybrid structures with spin filters as barriers, arbitrary values of the exchange field and arbitrary directions of the magnetization in the barrier and in the electrodes. We start with the model used in Ref.[36] and extend it to dissipative tunnel junctions. In the first part we focus on the study of the quasi-particle current and derive a general expression for the tunneling conductance. This expression recovers well-known results in particular limiting cases, and predicts effects related to the mutual orientation of the magnetizations. We study the tunneling conductance of different junctions such as $F/I_{sf}/F$, $HM/I_{sf}/HM$, and $F/I_{sf}/S$ (HM stands for a ferromagnetic half metal). In the second part we focus on the subgap transport through a $N/I_{sf}/S$ junction using quasiclassical formalism. In order to quantify the effect of spin-filtering on the proximity effect we need to generalize the existing boundary conditions[29,38] for the quasiclassical equations. Accordingly, we present a heuristic derivation of BCs which account for the spin-filter effect. These boundary conditions can be used in a wide range of problems involving superconductors, ferromagnets, and spin-filter tunnel barriers. As an example, we study the subgap conductance of an $N/I_{sf}/S$ junction and show its suppression due to the spin-filter effect. Thus, our work provides on the one hand a powerful tool for the theoretical study of spin transport in hybrid structures and on the other hand general expressions for the conductance that can be used for the interpretation of a broad range of experiments on spin transport through spin filters.

II. MODEL

We consider a generic tunnel junction $X_l/I_{sf}/X_r$ as the one shown in Fig. 1. The left and right electrodes, X_l and X_r are either a normal N metal, a ferromagnet F metal, a conventional superconductor S, or a superconductor S_M with a built-in exchange field h. The layer I_{sf} is a spin-filter barrier, *i.e.,* a spin-dependent tunneling barrier. Our first aim is to derive a general expression for the current in both structures. For this purpose, we use the well known tunneling Hamiltonian method, which has been used in several works on tunneling in superconducting junctions (see Refs.[39–42] and references therein) and in systems with charge- and spin-density waves[44,45]. The Hamiltonian consists of the Hamiltonians of the left (right) electrodes and the tunneling term

$$H = H_r + H_l + H_T \; . \tag{2.1}$$

For the electrodes we consider the general Hamiltonian

$$H_l = \sum_{\mathbf{p},s} \xi_{\mathbf{p}} a^\dagger_{\mathbf{p}s} a_{\mathbf{p}s} + \sum_{\mathbf{p}} \left(\Delta a^\dagger_{\mathbf{p}\uparrow} a^\dagger_{-\mathbf{p}\downarrow} + h.c. \right) - \sum_{\mathbf{p},s,s'} a^\dagger_{\mathbf{p}s} (h_l \mathbf{n}.\hat{\sigma})_{ss'} a_{\mathbf{p}s'}, \tag{2.2}$$

where $a_{\mathbf{p}s}(a^{\dagger}_{\mathbf{p}s})$ is the annihilation (creation) operator of a particle with momentum $\mathbf{p}$ and spin s, $\xi_{\mathbf{p}}$ is the quasiparticle energy, Δ is the superconducting gap, $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ is a vector with the Pauli matrices, h_l is the amplitude of the effective exchange field, and $\mathbf{n}$ is a unit vector pointing in its direction. A similar Hamiltonian can be written for the right electrode.

The H_T term in Eq. (2.1) describes the spin-selective tunneling through the spin-filter barrier I_{sf} and is given by

$$H_T = \sum_{\{n,s,\mathbf{p},\mathbf{p}'\}} \left(\mathcal{T}\hat{\sigma}_0 + \mathcal{U}\hat{\sigma}_3\right)_{ss'} a^{\dagger}_{\mathbf{p}s} b_{\mathbf{p}'s'} + h.c. \tag{2.3}$$

where a and b are the operators in the left and right electrodes, respectively; $\mathcal{T}$ and $\mathcal{U}$ are the tunneling spin-independent and spin-dependent matrix elements, respectively. For simplicity, we neglect the momentum dependence of $\mathcal{T}$ and $\mathcal{U}$, assuming that by tunneling the electrons are scattered randomly. The tunneling amplitude for spin up(down) is given by the relation: $T_{\uparrow(\downarrow)} = \mathcal{T} \pm \mathcal{U}$. The origin of the different tunneling amplitudes might be, for example, the conduction-band splitting in I_{sf}, which leads to different tunnel barrier heights for spin-up and spin-down electrons[23,24].

In order to write the equation of motions for the Green's functions it is convenient to write the tunneling Hamiltonian Eq.(2.3) in terms of new operators defined in an enlarged space (spin⊗particle hole). These operators are defined as:

$$A_{n,s} = \begin{cases} a_s & for \;\; n = 1 \\ a^{\dagger}_{\bar{s}} & for \;\; n = 2 \end{cases} \tag{2.4}$$

where $s = 1, 2$ is the spin-up (-down) index and $\bar{s}$ implies the change $1 \leftrightarrow 2$. In analogy one introduces the operators $B_{n,s}$ for the right electrode. By using these operators the tunneling term Eq.(2.3) can be written as

$$H_T = \sum_{\{n,s,\mathbf{p},\mathbf{p}'\}} [\check{A}^{\dagger}_{\mathbf{p}} \left(\mathcal{T}\hat{\tau}_3 \otimes \hat{\sigma}_0 + \mathcal{U}\hat{\tau}_0 \otimes \hat{\sigma}_3\right) \check{B}_{\mathbf{p}'} + h.c.] \tag{2.5}$$

where $\check{A} = A_{n,s}$ and $\check{B} = B_{n,s}$. The Hamiltonian Eq.(2.2) transforms to (see, for example, Ref.[19])

$$H_l = \sum_{\{n,s,\mathbf{p}\}} \check{A}^{\dagger}_{\mathbf{p}} \mathcal{H}_l \check{A}_{\mathbf{p}}, \tag{2.6}$$

with $\mathcal{H}_l = \xi_{\mathbf{p}}\hat{\tau}_3 \otimes \hat{\sigma}_0 + (\Delta\hat{\tau}_1 \otimes \hat{\sigma}_3 + h.c.) - h_l\hat{\tau}_0 \otimes \hat{\sigma}_3$, for an exchange field directed along the z axis. We assume throughout this work that the transport takes place in the x direction while the magnetization vector of the ferromagnets lies in the junction plane, i. e., the $y-z$-plane (see Fig. 1). One can take into account an arbitrary direction of the exchange field (or magnetization vector[49]) by means of the following rotation in spin space

$$\check{R}_{\alpha} = \cos(\alpha/2) + i\hat{\tau}_3 \otimes \hat{\sigma}_1 \sin(\alpha/2) \;, \tag{2.7}$$

where α is the rotation angle.

In order to calculate the tunneling current through the generic junction, we first write the Dyson equation for the Keldysh Green's functions $\check{G}_l$, for instance, in the left electrode

$$i\partial_t \check{G}_l = 1 + \check{\Sigma}_l \check{G}_l + \check{\mathcal{H}}_l \check{G}_l \tag{2.8}$$

Here $\check{\Sigma}_l = \sum_{\mathbf{q}} \check{\Gamma}\check{G}_r(\mathbf{q})\check{\Gamma}^{\dagger}$ is the self-energy part[39,43–46] related to the tunnel Hamiltonian Eq.(2.5), $\check{G}_r$ is the full (non-quasiclassical) Green's function in the right electrode, and $\check{\Gamma} = \mathcal{T}\hat{\tau}_3 \otimes \hat{\sigma}_0 + \mathcal{U}\hat{\tau}_0 \otimes \hat{\sigma}_3$. In the following we restrict our analysis to the lowest order in tunneling. In this case, the Green's function $\check{G}_l$ is determined by Eq.(2.8) after neglecting the second term on the right-hand side (rhs). The exact form of $\check{G}_l$ is given in Eq.(3.1).

We proceed as usual, and subtract from Eq.(2.8) (prelimenary multiplied by $\hat{\tau}_3$ from the left) its conjugated equation multiplied by $\hat{\tau}_3$ from from the right:

$$i(\hat{\tau}_3\partial_t \check{G} + \partial_{t'}\check{G}\hat{\tau}_3)_l = \hat{\tau}_3\check{\Sigma}\check{G} - \check{G}\check{\Sigma}\hat{\tau}_3 + \hat{\tau}_3\check{G}\check{\mathcal{H}}_l - \mathcal{H}_l\check{G}\hat{\tau}_3|_l \;. \tag{2.9}$$

If we now multiply the Keldysh component of this equation by the electron charge e, set $t = t'$, take the trace and sum up over momenta, we obtain on the left-hand side the time derivative of the charge, $\partial_t Q$. Thus, the current density j through the barrier is determined by the first two terms on the rhs of Eq. (2.9)

$$I_T = -\frac{e}{32\pi} \sum_{\mathbf{p},\mathbf{q}} \int \frac{d\epsilon}{2\pi} \mathrm{Tr}\left\{\hat{\tau}_3 \otimes \hat{\sigma}_0 \left[\check{\Gamma}_{\alpha\beta}\check{G}_r(\mathbf{q},\epsilon)\check{\Gamma}^{\dagger}_{\alpha\beta}, \check{G}_l(\mathbf{p},\epsilon)\right]^K\right\} \tag{2.10}$$

where $\check{\Gamma}_{\alpha\beta} \equiv \check{R}_\alpha \check{\Gamma} \check{R}^\dagger_\beta$, and $\check{\Gamma}$ as $\check{\Gamma} \equiv \check{\Gamma}\hat{\tau}_3 \otimes \hat{\sigma}_0 = \mathcal{T} + \mathcal{U}\hat{\tau}_3 \otimes \hat{\sigma}_3$. The Green's functions $\check{G}_{l,r}$ correspond to the case of the magnetization vector oriented parallel to the z-axis. For arbitrary magnetization orientation one can express the Green's functions $\check{G}_{l\alpha}(\mathbf{p},\epsilon)$ through the matrices $\check{G}_{l,r}$ withe the help of the rotation Eq. (2.7): $\check{G}_{l\alpha}(\mathbf{p},\epsilon) = \check{R}_\alpha \int d(t-t')$ $\check{G}_{l0}(\mathbf{p}, t-t') \exp(i\epsilon(t-t'))\check{R}^\dagger_\alpha$.

In the case that the energies involved in the problem are much smaller than the Fermi energy, one can perform the momentum integration in Eq.(2.10) and the current can be written in terms of quasiclassical Green's functions $\check{g}_{l\alpha} = (i/\pi)\hat{\tau}_3 \otimes \hat{\sigma}_0 \int d\xi_p \check{G}_{l\alpha}(\mathbf{p},\epsilon)$

$$I_T R_N = [16e(\mathcal{T}^2 + \mathcal{U}^2)]^{-1} \int d\epsilon \mathrm{Tr}\left\{\hat{\tau}_3 \otimes \hat{\sigma}_0 \left[\check{\Gamma}_{\alpha\beta}\, \check{g}_r(\epsilon) \check{\Gamma}^\dagger_{\alpha\beta}, \check{g}_l(\epsilon)\right]^K\right\} \tag{2.11}$$

The resistance $R_N = [4\pi e^2 N_l(0) N_r(0)(\mathcal{T}^2 + \mathcal{U}^2)]^{-1}$ is the junction resistance in the normal state, i. e., the resistance of an $F/I_{sf}/F$ junction with parallel orientation of magnetization along the z axis, $N_{l,r}(0) = (p_F m/2\pi^2)_{l,r}$ are the density of states (DOS) at the Fermi level. One should have in mind that by going over to the quasiclassical Green's functions we lose the spin dependence of the DOS in the normal state. In that case the retarded (advanced) Green's functions $\check{g}^{R(A)}_{l,r}$ in the ferromagnet have a trivial spin structure, $\check{g}^{R(A)}_F = \pm\hat{\tau}_3 \otimes \hat{\sigma}_0$, so that the normalized density of states is the same for spin up and down. This approach is valid for electrodes with small spin-splitting at the Fermi level, and was used for example in Ref.[36] for the calculation of the Josephson current through a $S/I_{sf}/S$ junction. However, if the spin-polarization of the electrodes at the Fermi level is large enough one has to use Eq.(2.10) in order to compute the current. This is done in the next section, in which we calculate the conductance of a $F/I_{sf}/S_M$ and a $F/I_{sf}/F$ junction.

III. THE CONDUCTANCE FOR JUNCTIONS WITH ARBITRARY EXCHANGE FIELDS

In this section we consider junctions of the type $F/I_{sf}/F$ and $F/I_{sf}/S_M$, where F is a ferromagnet and S_M describes either a thin FS bilayer[47] or a superconductor with an induced spin-splitting field due to the proximity of the magnetic barrier I_{sf}[48]. We are interested in arbitrary strength of the exchange field and therefore we have to go beyond quasiclassics and use Eq. (2.10) for the current. We assume a bias voltage V between the electrodes setting the electric potential in the superconducting electrode equal to zero (Fig. 1). In the tunneling limit the junction under consideration can only carry a normal (quasiparticle) current, which is determined by the normal Green's functions $\check{G}_{l,r}$. These are diagonal in spin space with diagonal elements given by

$$\hat{G}^R_{r\pm}(\mathbf{p}) = \frac{(\epsilon_\pm + i\gamma)\hat{\tau}_0 + \xi_\mathbf{p}\hat{\tau}_3}{(\epsilon_\pm + i\gamma)^2 - (\xi^2_\mathbf{p} + \Delta^2)}\ , \tag{3.1}$$

where $\epsilon_\pm = \epsilon \pm h_r$, $\xi_\mathbf{p} = (p^2 - p^2_{F,r})/2m_r$ and γ is a damping in the excitation spectrum of the superconductor due to inelastic processes or due to coupling with the normal metal electrode. The corresponding Green's function in the left (F) electrode has the same form if we set $\Delta = 0$ and replace the index r by l. As usual, the advanced Green's function G^A is defined in a similar way with the opposite sign of the damping term, $-i\gamma$. The full Green's function in a superconductor has the form $\check{G}^R_r = \hat{G}^R_r \otimes \hat{\tau}_0 + \hat{F}^R_r \otimes \hat{\tau}_1$, where $\hat{G}^R_r = G^R_{r0}\hat{\sigma}_0 + G^R_{r3}\hat{\sigma}_3$, $G^R_{r0,3} = (G^R_{r+} \pm G^R_{r-})/2$, and $\hat{F}^R_r$ is the anomalous (Gor'kov's) Green's function. Using the fact that the normal parts of matrices $\check{G}_{r,l}$ are diagonal in the spin and particle-hole space, we can represent the current, Eq. (2.10), in the form

$$I_T = \frac{\pi e}{2}[(N_{n\uparrow} + N_{n\downarrow})_l (N_{n\uparrow} + N_{n\downarrow})_r] \int d\epsilon n_V(\epsilon) \mathrm{Tr}\left\{\hat{\tau}_0 \otimes \hat{\sigma}_0 \left[\check{\Gamma}_{\alpha\beta}\, \check{\nu}_l(\epsilon) \check{\Gamma}^\dagger_{\alpha\beta}\, \check{\nu}_r(\epsilon)\right]\right\}\ , \tag{3.2}$$

where $n_V(\epsilon) = \{\tanh[(\epsilon + eV)/2T] - \tanh[(\epsilon - eV)/2T]\}/2$, and T is the temperature. The matrices $\check{\nu}_{l,r}(\epsilon)$ are related to $\hat{G}^R_{r\pm}(\mathbf{p})$ via $\check{\nu}(\epsilon) = (i/2\pi)(N_{n\uparrow} + N_{n\downarrow})^{-1} \sum_\mathbf{p} [\check{G}^R(\mathbf{p}) - \check{G}^A(\mathbf{p})]$, and can be written in the form: $\check{\nu}_{l,r\pm}(\epsilon) = [\nu_0(\epsilon)\hat{\tau}_0 \otimes \hat{\sigma}_0 + \nu_3(\epsilon)\hat{\tau}_3 \otimes \hat{\sigma}_3]_{l,r}$ with $\hat{\nu}_{0,3}(\epsilon) = [\hat{\nu}(\epsilon + h) \pm \hat{\nu}(\epsilon - h)]/2$. It is useful to write the coefficients $\nu_{0,3}(\epsilon)$ in terms of the DOS for spin up and down, $N_{\uparrow,\downarrow}$: $\nu_{0,3} = (N_\uparrow \pm N_\downarrow)/(N_{n\uparrow} + N_{n\downarrow})$, where $N_{n\uparrow,\downarrow}$ are the DOS at the Fermi level in the normal state of the ferromagnets. The matrices $\check{\Gamma}_{\alpha\beta}$ describe the tunneling probability and are given by

$$\check{\Gamma}_{\alpha\beta} = \mathcal{T}\hat{\tau}_3 \otimes \hat{\sigma}_0 \cos(\frac{\alpha-\beta}{2}) + \mathcal{U}\hat{\tau}_0 \otimes \hat{\sigma}_3 \cos(\frac{\alpha+\beta}{2}) + i\mathcal{T}\hat{\tau}_0 \otimes \hat{\sigma}_1 \sin(\frac{\alpha-\beta}{2}) - \mathcal{U}\hat{\tau}_3 \otimes \hat{\sigma}_2 \sin(\frac{\alpha+\beta}{2}) \tag{3.3}$$

By substituting these expressions into Eq.(3.2), we find for the normalized conductance

$$G_{\alpha\beta}(V) \equiv R_F dI_T/dV = (1/2e) \int d\epsilon (dn_V/dV) Y_{\alpha\beta}(\epsilon)\ , \tag{3.4}$$

where the spectral conductance $Y_{\alpha\beta}(\epsilon)$ is defined as

$$Y_{\alpha\beta}(\epsilon) = \{\nu_{0l}\nu_{0r} + \frac{[\mathcal{T}^2\cos(\alpha-\beta)+\mathcal{U}^2\cos(\alpha+\beta)]}{(\mathcal{T}^2+\mathcal{U}^2)}\nu_{3l}\nu_{3r} + 2\frac{\mathcal{T}\mathcal{U}}{(\mathcal{T}^2+\mathcal{U}^2)}[\nu_{0l}\nu_{3r}\cos\beta + \nu_{3l}\nu_{0r}\cos\alpha]\}\ . \tag{3.5}$$

and R_F is defined as $R_F = [\pi e^2(N_{n\uparrow}+N_{n\downarrow})_l(N_{n\uparrow}+N_{n\downarrow})_r]^{-1}$.

Equations (3.4-3.5) comprise one of the main results of the present paper. They determine the conductance of the generic junction of Fig. 1 in a quite general situation, because they are valid for the arbitrary exchange field, spin-filter strength and angles α and β. At low temperatures ($T \ll \Delta$) one can evaluate the energy integral in Eq. (3.4) obtaining a simple expression for the normalized conductance

$$G_{\alpha\beta}(V) = \frac{Y_{\alpha\beta}(eV) + Y_{\alpha\beta}(-eV)}{2}\ . \tag{3.6}$$

We now proceed to consider different types of junctions and calculate the conductance with the help of Eqs. (3.4-3.6)

A. Junctions with non-superconducting electrodes

Let us first consider junctions in the normal state. For example in a $N/I_{sf}/F$ junction the left electrode is a normal metal with no spin-polarization, therefore $\nu_3 = 0$ and $\nu_0 = 1$. From Eq.(3.5) we then obtain

$$Y^{(NF)}_{\alpha\beta} = 1 + \rho_b P_r \cos\beta\ , \tag{3.7}$$

where we have defined the polarization of the electrodes as $P_{l(r)} = (N_{\uparrow,l(r)} - N_{\downarrow,l(r)})/(N_{\uparrow,l(r)} + N_{\downarrow,l(r)})$. We have also introduced the quantity $\rho_b = 2\mathcal{T}\mathcal{U}/((\mathcal{T}^2+\mathcal{U}^2)) = (T_\uparrow^2 - T_\downarrow^2)/(T_\uparrow^2 + T_\downarrow^2)$, which is a measure for the spin-filter efficiency. The quantity ρ_b equals zero for the spin-independent transmission coefficient and equals one for a 100% spin-filter effect. Equation (3.7) shows that in the presence of a spin-filtering effect ($\rho_b \neq 0$), the conductance depends on the relative angle β between the magnetizations of the F electrode and the spin-filter barrier.

In the case of a $F/I_{sf}/F$ junction, we obtain a general expression for the spectral conductance

$$Y^{(FF)}_{\alpha\beta} = 1 + P_l P_r \frac{[\mathcal{T}^2\cos(\alpha-\beta)+\mathcal{U}^2\cos(\alpha+\beta)]}{(\mathcal{T}^2+\mathcal{U}^2)} + \rho_b[P_r\cos\beta + P_l\cos\alpha]\ , \tag{3.8}$$

In order to make a connection with the effect of tunnel magnetoresistance we define the relative conductance change as

$$TMR = \frac{G_{00} - G_{0\pi}}{G_{0\pi}}\ . \tag{3.9}$$

Thus, for the $F/I_{sf}/F$ junction one obtains

$$TMR = 2\frac{P_r(P_l+\rho_b)}{1 - P_r(P_l+\rho_b) + \rho_b P_l}\ . \tag{3.10}$$

If we assume that there is no spin filter, *i.e.* $\rho_b = 0$ then Eq. (3.10) leads to the well-known Julliere's formula[63].

Now we consider ferromagnetic electrodes with collinear magnetizations. We distinguish two magnetic configurations: the parallel one **P**, i.e $\beta = \alpha$ and the antiparallel configuration **AP**, $\beta = \alpha + \pi$. In Fig.(2), we show the conductance of a junction with $P_l = P_r$ in these two cases. In the **P** configuration the conductance is 2π periodic, and, depending on the angle α, it is larger or smaller than in the non magnetic case. The largest value of the conductance is obtained when the magnetizations of the electrodes and the barrier are parallel. In the **AP** configuration the conductance is π periodic and it is always smaller than in the non magnetic case. It is interesting to note that in the case of a fully polarizing barrier ($\rho_b = 1$) and perpendicular magnetization of the ferromagnets with respect to I_{sf} ($\alpha = \pi/2$), the normalized conductance equals 1 for all values of the spin polarization of the electrodes and for both configurations **P** and **AP** [see Figs. 2(b) and 2(d)].

If the junction consists of two half-metallic ferromagnets (HM) then $P_{l,r} = 1$ and from Eq. (3.8) one obtains

$$Y^{(HM)}_{\alpha\beta} = 1 + \frac{[\mathcal{T}^2\cos(\alpha-\beta)+\mathcal{U}^2\cos(\alpha+\beta)]}{(\mathcal{T}^2+\mathcal{U}^2)} + \rho_b[\cos\beta + \cos\alpha]\ , \tag{3.11}$$

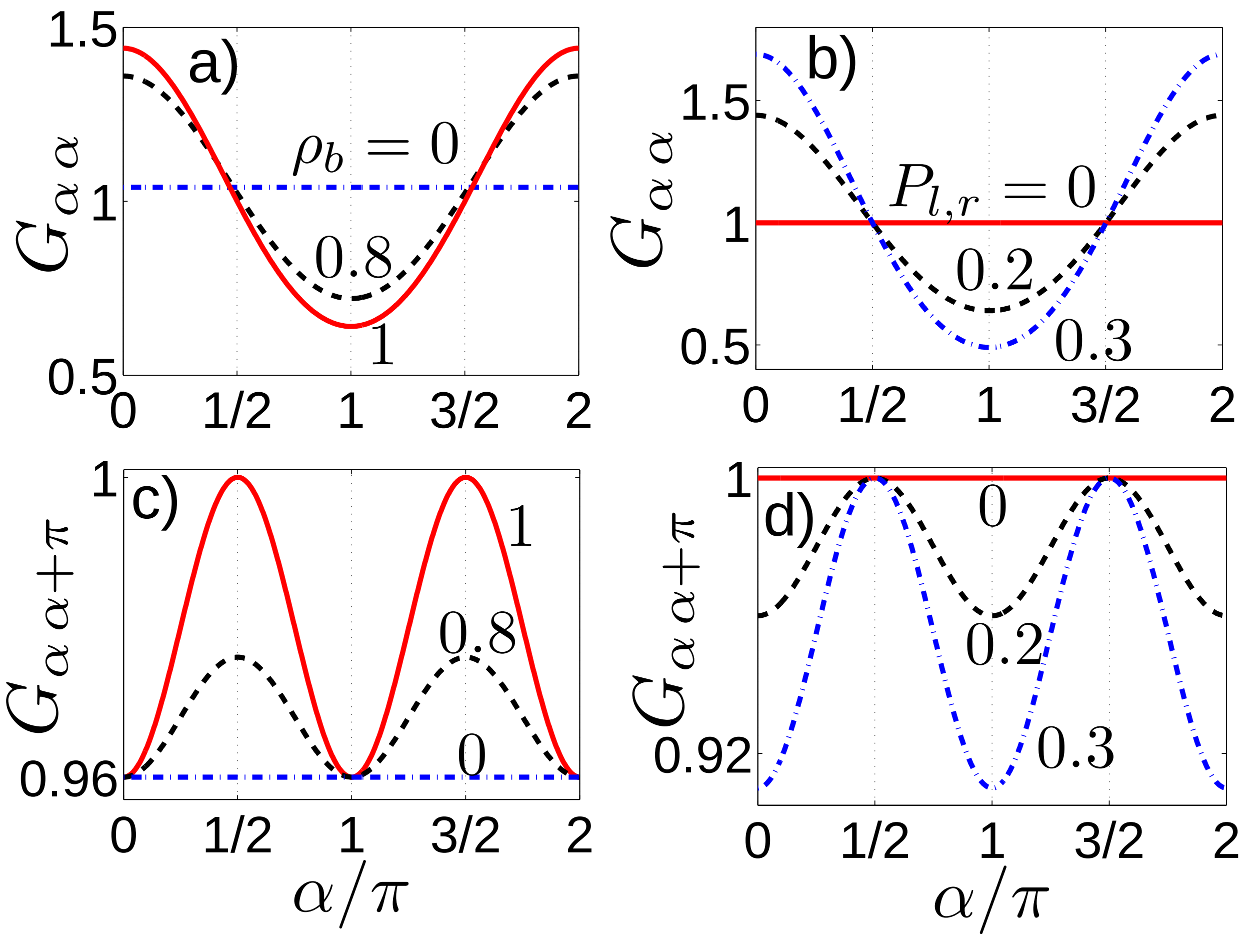

FIG. 2: The normalized conductance $G_{\alpha,\beta}$ for a $F_l/I_{sf}/F_r$ junction as a function of the magnetization orientation with respect to the spin-filter. Panels (a) and (b) are for the parallel (**P**) configuration, (i.e $\beta = \alpha$); whereas panels (c) and (d) are for the antiparallel (**AP**) configuration ($\beta = \alpha + \pi$). In panels (a) and (c), we set $P_l = P_r = 0.2$ and vary $\rho_b = 0, 0.8, 1$. In panels (b) and (d), we choose $\rho_b = 1$ and vary $P_r = P_l = 0, 0.2, 0.3$.

It is clear from this expression that the conductance of a non-magnetic barrier ($\mathcal{U} = 0$) vanishes if the magnetizations of the left and right electrodes are antiparallel ($\beta = \alpha + \pi$).

If $\alpha = \beta = 0$ and $\alpha = \beta = \pi$, the spectral conductance is given by

$$Y^{(HM)}_{00,\pi\pi} = 2(1 \pm \rho_b) \tag{3.12}$$

As expected, the conductance of the $HM/I_{sf}/HM$ junction with a barrier that is impenetrable for one spin direction vanishes if the magnetization in the left and right electrodes are antiparallel with respect to the magnetization of the barrier.

B. Junctions with one superconducting electrode

We consider now a $F/I_{sf}/S$ junction and calculate the conductance of the system using Eqs. (3.4-3.5). In the superconducting electrode $\nu_{0,3,r}(\epsilon) = [\nu_r(\epsilon + h_r) \pm \nu_r(\epsilon - h_r)]/2$, where $\nu_r(\epsilon) = \epsilon/\sqrt{\epsilon^2 - \Delta^2}$. Here h_r is an effective exchange field induced in the superconductor by the proximity of a thin F-layer (as in a $F/I_{sf}/FS$ junction) or by the proximity of the magnetic barrier I_{sf} itself[48]. In this case ν_{0l}, ν_{0r} and ν_{3l} are even functions of ϵ and ν_{3r} is an

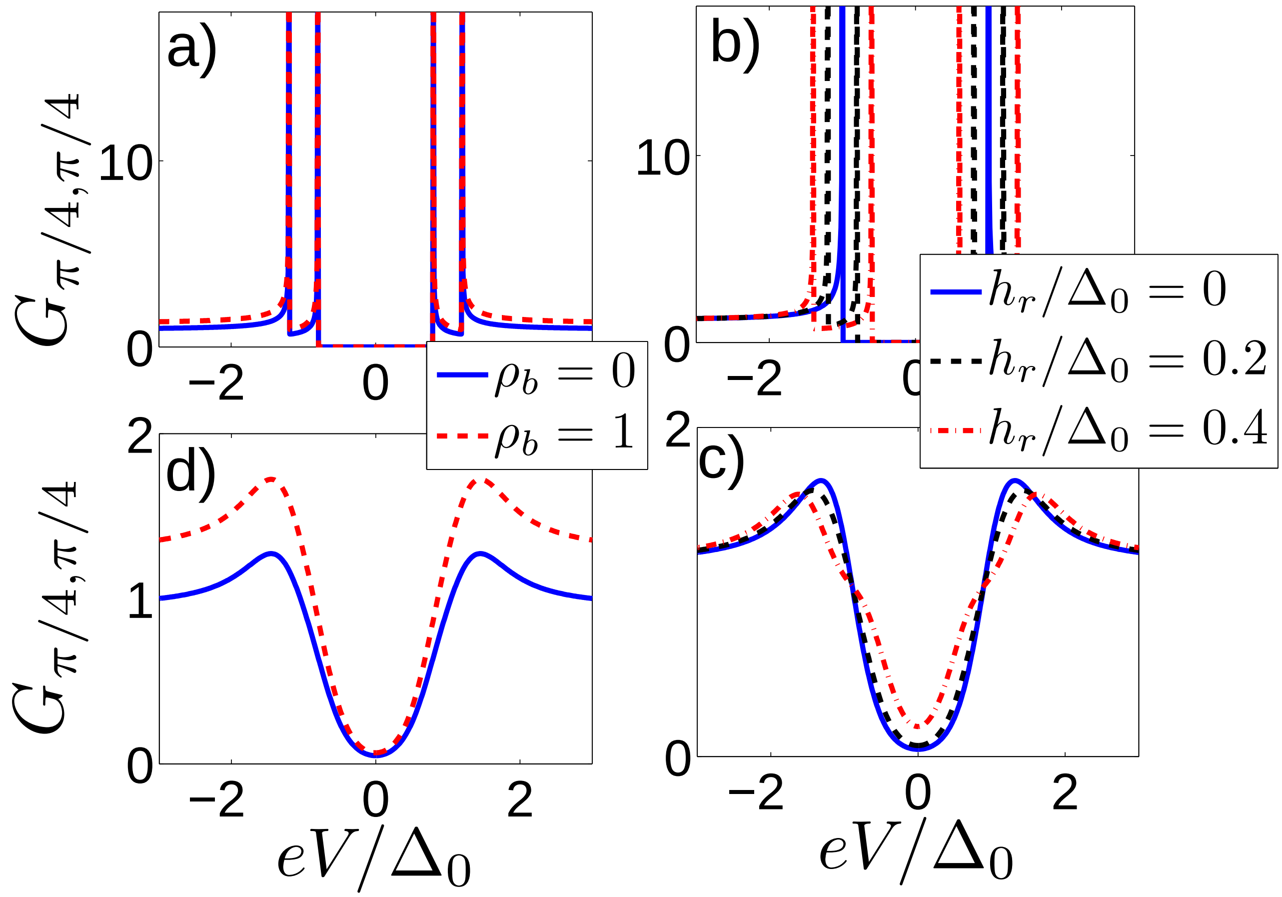

FIG. 3: The voltage dependence of the normalized conductance $G_{\pi/4,\pi/4}$ for a $F/I_{sf}/S_M$ junction with fixed polarization in the ferromagnetic electrode ($P_l = 0.5$) and fixed angles ($\alpha = \beta = \pi/4$). Panels (a) and (b) are for zero temperature, whereas panels (c) and (d) are for $T = 0.2T_c^0$, where T_c^0 is the critical temperature at zero exchange field. In panels (a) and (c) we set $h_r = 0.2\Delta_0$ and vary ρ_b. In panels (b) and (d) we set $\rho_b = 0.8$ and vary h_r. By calculating the curves we set the damping factor $\gamma = 0.01\Delta_0$.

odd functions of ϵ (in the quasiclassical approximation). Only the first and last terms of Eq. (3.5) contribute to the integral in Eq. (3.4). Thus, the spectral conductance $Y_{\alpha\beta}(\epsilon)$ can be written as follows

$$Y_{\alpha\beta}(\epsilon)^{(FS)} = \nu_{0r}\left(1 + \rho_b P_l \cos\alpha\right) \ . \tag{3.13}$$

This equation, as well as Eq. (3.5), resembles the Slonczewski formula[64]. It generalizes the latter for the case of a superconducting electrode and a spin-filter barrier.

In Fig.3 we show the conductance for the $F/I_{sf}/S$ junction obtained from Eqs. (3.4, 3.13). One can see the splitting of the conductance peaks at $eV = \pm\Delta \pm h_r$ due to the finite exchange field h_r in the superconducting electrode. Note that by increasing the temperature the peaks smeared out [see Figs. 3(c) and 3(d)]. From Figs. 3(a) and 3(c), one can see that the values of G in the normal state, *i.e.* the asymptotic values for $V \gg \Delta$, depends on the polarization of the barrier, in accordance with Eq. (3.7).

IV. SUBGAP CONDUCTANCE IN $N/I_{sf}/S$ JUNCTIONS: EFFECTIVE BOUNDARY CONDITIONS

In the previous sections, we have calculated the conductance of different junctions in the tunneling limit. In other words, the Green's functions in the left and right electrodes have not been corrected due to the proximity effect. However, it is well known that the proximity effect in N/S structures induces a condensate in the normal metal which causes a subgap conductance in N/S junctions[53,54]. In order to quantify the proximity effect we need boundary conditions that take into account the spin filtering at the N/S barrier. Surprisingly, in spite of many works on N/S and F/S structures such boundary conditions are absent in the literature[25–31,56,57]. In order to fill, this gap we present here a heuristic derivation of the boundary conditions at the $S_M/I_{sf}/S_M$ interface, which can also be used for $N/I_{sf}/S$ or $S-I_{sf}-F/S$ interfaces.

We consider the diffusive limit and write down the Usadel-like equation for the Keldysh function $\check{g}^K$ in the S_M electrodes

$$-D\partial_x(\check{g}\partial_x\check{g})^K - [i\hat{\tau}_3\otimes(\hat{\sigma}_0\epsilon - \hat{\sigma}_3 h(x)) + \Delta\hat{\tau}_2\otimes\hat{\sigma}_3, \check{g}^K] = 0 \tag{4.1}$$

We assume that the exchange field h differs from zero only in a thin enough layer of thickness $d_F \ll \xi_h$. This allows us to integrate Eq.(4.1) over the thickness d_F considering the Green's functions $\check{g}^K$ to be constant in this narrow layer. Performing this procedure at $x>0$, we obtain for the "spectral" matrix current $\check{I}^K(\epsilon)$

$$\check{I}^K(\epsilon) \equiv (\check{g}\partial_x\check{g})^K|_{x=d_F} = i\kappa_h[\hat{\tau}_3\otimes\hat{\sigma}_3, \check{g}_{r(l)}] + \check{I}^K_T \tag{4.2}$$

where $\kappa_h = hd_F/D$. The latter term describes the tunneling current. The charge current density, for example, in the right superconductor is given by

$$I = (\sigma_r/16e)\int d\epsilon \mathrm{Tr}\left\{\hat{\tau}_3\otimes\hat{\sigma}_0[\check{g}_r\partial_x\check{g}_r]^K\right\} \tag{4.3}$$

This current equals the tunneling current given in Eq.(2.11). Therefore one can assume that

$$\check{I}^K_T = \frac{\kappa_T}{\mathcal{T}^2+\mathcal{U}^2}\left[\check{\Gamma}_{\alpha\beta}\,\check{g}_l\check{\Gamma}^\dagger_{\alpha\beta}, \check{g}_r\right]^K \tag{4.4}$$

where $\kappa_T = 1/(\sigma_r R_{N\Box})$, σ_r is the conductivity of the right S electrode in the normal state, and $R_{N\Box}$ is the interface resistance in the normal state per unit area.

Equations (4.2) and (4.4) represent the boundary conditions (BC) for the Keldysh matrix function $\check{g}^K_l$. Equivalent equations hold for the retarded (advanced) Green's functions, $\check{g}^{R(A)}$, if the index K is replaced by indices $R(A)$. We can then write a boundary condition for the matrix Green's function $\check{g}$ in a general form

$$(\check{g}_{r(l)}\partial_x\check{g}_{r(l)})|_{x=0} = i\kappa_h[\hat{\tau}_3\otimes\hat{\sigma}_3, \check{g}_{r(l)}] + \frac{\kappa_T}{\mathcal{T}^2+\mathcal{U}^2}\left[\check{\Gamma}_{\alpha\beta}\,\check{g}_{l(r)}\,\check{\Gamma}^\dagger_{\alpha\beta}, \check{g}_{r(l)}\right]\ . \tag{4.5}$$

This condition generalizes the Kuprianov-Lukichev (K-L) BCs[38] for the case of spin-dependent transmission coefficients and in the presence of an effective exchange field h[65]. Equation (4.5) is valid for the case in which the tunneling matrix elements $\mathcal{T}_\uparrow$ and $\mathcal{T}_\downarrow$ do not depend on momenta. In other words, no component of the momentum is conserved by tunneling (diffusive interface). The physical meaning of the BCs in Eq.(4.5) is rather simple. The first term stems from the finite exchange field in the vicinity of the I_{sf}/S interface, while the second term on the r.h.s. is due to the tunneling through the barrier with spin-dependent transmission coefficients $\mathcal{T}_{\uparrow(\downarrow)}$. Note that in equilibrium, Eq. (4.5) is also valid for the Matsubara Green's functions $\check{g}_\omega$.

We emphasize that the above derivation of the BC Eq. (4.5) cannot be regarded as a microscopic derivation. However, these BCs give correct physical results, and can be used, for example, to calculate the tunnel current in $S_M/I_{sf}/S_M$ junctions and for the study of the proximity effect in I_{sf}/S_M and other systems.

One can compare the BCs in Eqs.(4.5) with those obtained earlier for diffusive systems[29,38]. In the nonmagnetic case, i. e., when the matrix $\check{\Gamma}_{\alpha\beta}$ is a scalar Γ and $h=0$, Eq.(4.5) coincides with the K-L BC[38]. If $h\neq 0$, the first term on the rhs of Eq. (4.5) coincides with the third term in the r.h.s. of Eq.(61) of Ref.[29]. Moreover, If we assume that the magnetization vectors in the superconductors are parallel to the z-axis then $\check{\Gamma}_{00} = \mathcal{T} + \mathcal{U}\hat{\tau}_3\otimes\hat{\sigma}_3$ and we obtain:

$$\left[\check{\Gamma}_{00}\,\check{g}_l\check{\Gamma}^\dagger_{00}, \check{g}_r\right] = \mathcal{T}^2\left[\check{g}_l,\check{g}_r\right] + \mathcal{U}^2\left[\hat{\tau}_3\otimes\hat{\sigma}_3\check{g}_l\hat{\tau}_3\otimes\hat{\sigma}_3, \check{g}_r\right] + \mathcal{T}\mathcal{U}[\{\hat{\tau}_3\otimes\hat{\sigma}_3, \check{g}_l\}, \check{g}_r]\ . \tag{4.6}$$

We see that the last term proportional to $\mathcal{T}\mathcal{U}$ corresponds to the second term on the rhs of Eq.(61) of Ref.[29]. However, as it was shown in our previous work,[36] this term does not contribute to the Josephson current. The first correction

to the current due to the spin filter is of the order $\mathcal{U}^2$ and described by the second term in Eq. (4.6). The latter was neglected in Ref.[29]. This term is essential if one needs to describe the spin-filtering effect. Only due to this term the Josephson current is zero if either $\mathcal{T}_\uparrow$ or $\mathcal{T}_\downarrow$ is zero[36]. Notice that the BC condition derived in Ref.[29] contains other terms which are product of three Green's functions, i.e. they are higher order terms in the expansion with respect to the tunneling coefficients $\mathcal{T}$ and $\mathcal{U}$. The BCs in Eqs.(4.2,4.4) also describe an interface between different materials with, for example, different effective masses. In two recent works[61],[62] the BCs at an interface between different materials in a ballistic case were derived using another approach.

As an example, we use the derived boundary conditions in Eqs.(4.2,4.4) to study the proximity effect in a simple $N/I_{sf}/S$ system with a spin-filtering barrier. We assume a weak proximity effect and hence a small amplitude of the condensate function $\check{f}_N$ induced in the normal metal. We then can write $\check{g}_N = \hat{\tau}_3 \otimes \hat{\sigma}_0 + \check{f}_N$. The linearized BC (4.2) acquires the form

$$\partial_x \check{f}_N|_{x=0-} = -\frac{\kappa_T}{\mathcal{T}^2+\mathcal{U}^2}\check{\Gamma}_{00}\check{f}_S\check{\Gamma}^\dagger_{00} = -r\kappa_T f_S. \tag{4.7}$$

where $\check{f}_S^{R(A)} = \hat{\tau}_2 \otimes \hat{\sigma}_3 f_S^{R(A)}$, $f_S^{R(A)} = \Delta/\sqrt{\Delta^2-(\epsilon \pm i\gamma)^2}$ is the amplitude of the quasiclassical anomalous (Gor'kov's) Green's function in the S superconductor and $\check{\Gamma}_{00} = \mathcal{T} + \mathcal{U}\hat{\tau}_3 \otimes \hat{\sigma}_3$. We have defined the spin-filter parameter as $r = (2\mathcal{T}_\uparrow\mathcal{T}_\downarrow)/(\mathcal{T}_\uparrow^2+\mathcal{T}_\downarrow^2)$. The latter is related to the spin-filter efficiency ρ_b of the spin-filter barrier by the expression $r = \sqrt{1-\rho_b^2}$. For a barrier transparent only for one spin direction $r = 0$, while for a non-magnetic one $r = 1$.

The condensate in the normal metal has the same matrix structure as in S, $\check{f}_N = \hat{\tau}_2 \otimes \hat{\sigma}_3 f_N$, where the amplitude f_N is found from the linearized Usadel equation

$$\partial^2_{xx} f_N - \kappa_\epsilon^2 f_N|^{R(A)} = 0 \tag{4.8}$$

complemented with the BC Eq.(4.7). The solution of Eq.(4.8) can be easily written

$$f_N(x) = r(\kappa_T/\kappa_\epsilon) f_S \exp(\kappa_\epsilon x) \;, \tag{4.9}$$

where $\kappa_\epsilon^2|^{R(A)} = \mp 2i\epsilon/D$. Thus, the amplitude f_N of the induced condensate is proportional to the spin-filter parameter r. In particular the proximity effect is completely suppressed if $r = 0$. Although this result is quite obvious, it has not been obtained in any previous work.

We now consider the case in which a voltage difference V is applied to the $N/I_{sf}/S$ junction. Such a situation was studied both experimentally[53,5466] and theoretically[50–52,58–60]. It was observed that a zero-bias peak arises in the voltage dependence of the conductance. The origin of this peak is the induced condensate in the normal metal. In this case, the tunnel current consists not only of the usual quasiparticle current, but also of the current which is proportional to the product of the condensate amplitudes in S and N electrodes and to the applied voltage V.

In the presence of a spin-filter barrier, the subgap current can be obtained from Eq.(2.10)

$$I_{sg} = (32eR_N)^{-1}\int d\epsilon \mathrm{Tr}\left\{n_V \hat{\tau}_0 \otimes \hat{\sigma}_0 [(\check{f}_l^R + \check{f}_l^A)(\check{f}_r^R + \check{f}_r^A)]\right\} \;. \tag{4.10}$$

At low energies ($\epsilon \sim eV \ll \Delta$) one has $\check{f}_r^R \approx \check{f}_r^A \approx -\hat{\tau}_2 \otimes \hat{\sigma}_3$, $\check{f}_l^{R(A)} \approx -r[(\kappa_T \xi_T)(1 \pm i)\sqrt{2T/|\epsilon|}]\hat{\tau}_2 \otimes \hat{\sigma}_3$, $\xi_T = \sqrt{D/(2T)}$. For the normalize differential conductance, $G_{sg} = R_N dI_{sg}/dV$, we obtain

$$G_{sg} = \sqrt{1-\rho_b^2}(\kappa_T \xi_T/16)J(V) \;, \tag{4.11}$$

where we have used the fact that $r = \sqrt{1-\rho_b^2}$. Here $J(V) = \int d\epsilon[dn_V/d(eV)]\sqrt{2T/\epsilon}$. For $V = 0$ one has $J(0) \approx 3.41$, whereas in the limiting case of low temperatures ($V \gg T$), one obtains $J(V) \approx \sqrt{2T/eV}$. It is clear from Eq. (4.11) that spin-filtering suppresses the subgap conductance. In Fig.(4), we plot the voltage dependence of the normalized differential conductance taking into account the contribution of the quasiparticle and subgap currents given in Eqs.(3.5) and (4.10), respectively.

V. CONCLUSIONS

We have studied the transport properties of a generic $X_l/I_{sf}/X_r$ junction with a spin-filter tunneling barrier I_{sf}. The electrodes $X_{l,r}$ can be a normal metal, a ferromagnet, or a superconductor with or without a built-in exchange field. We have derived a general expression for the tunneling conductance, Eq. (3.4), which is valid for arbitrary

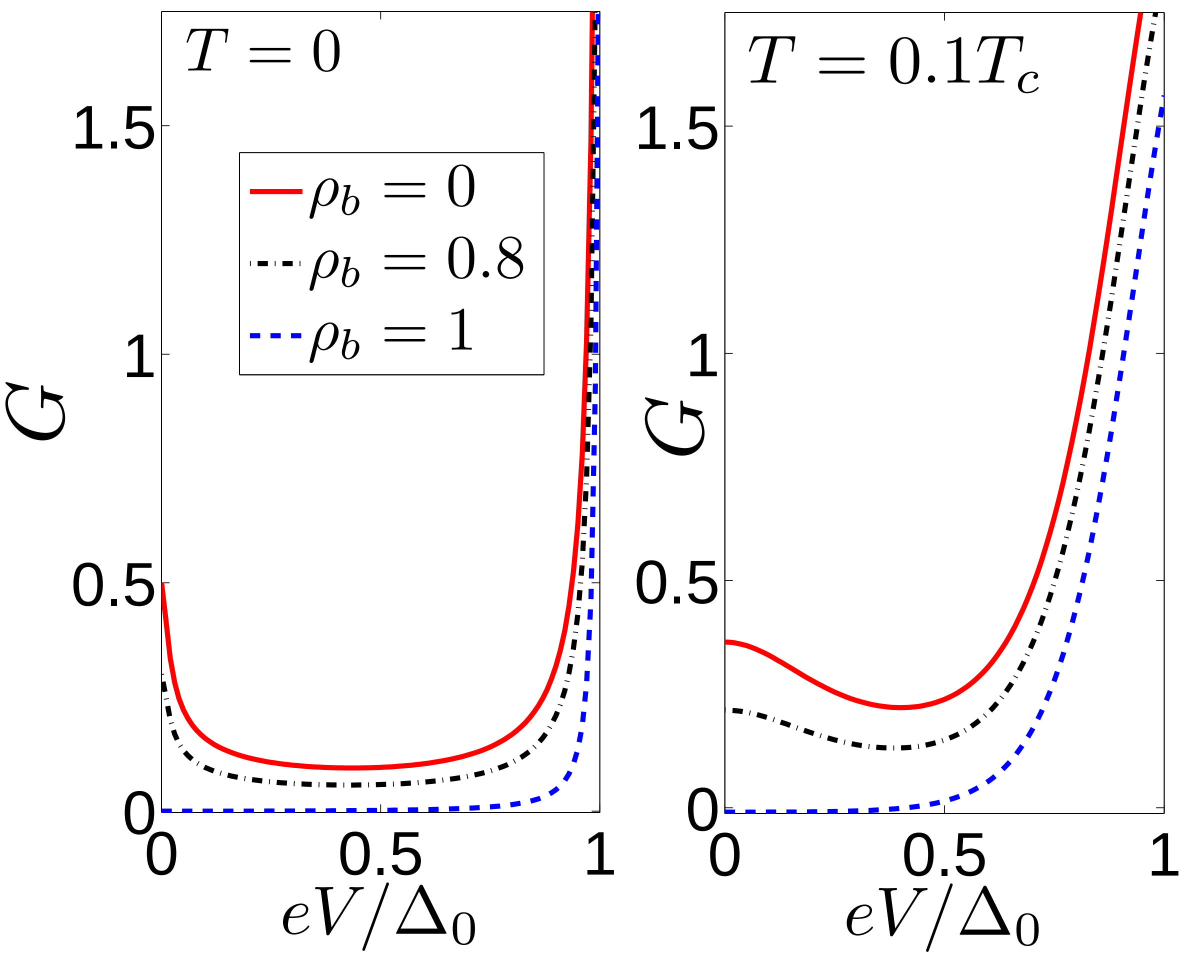

FIG. 4: The voltage dependence of the normalized conductance G, for a N/I$_{sf}$/S$_M$ junction, calculated from Eqs.(3.5) and (4.10). We set $\kappa_T\sqrt{D/\Delta_0} = 0.25$ and $T = 0$ in the left panel and $T/T_c = 0.1$ in the right one. By calculating the curves we have set the damping factor $\gamma = 0.01\Delta_0$.

values of the exchange fields and the angles between the magnetizations. This expression generalizes the well-known results for normal multilayer systems with collinear magnetization and shows how the conductance depends on the mutual orientation of the magnetization of the electrodes and the magnetic barrier, the spin-filter parameter and the spin-dependent density of the states in the normal and superconducting electrodes. We also have derived boundary conditions for the quasiclassical Green's functions, taking into account the spin filter effect. By using these boundary conditions we have studied the proximity effect in the $N/I_{sf}/S$ system and shown that spin filtering suppresses the amplitude of the condensate in the normal layer. In particular we show that the sub gap conductance of the $N/I_{sf}/S$ junction is suppressed due to the spin-filter effect by a factor of $\sqrt{1-\rho_b^2}$, where ρ_b is the spin-filter efficiency parameter.

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University.

[1] L.N. Bulaevskii, V.V. Kuzii, and A.A. Sobyanin, JETP Lett. **25**, 290 (1977) (Zh. Eksp.Teor. Fiz. **25**, 314 (1977)).
[2] A.I. Buzdin, L.N. Bulaevskii, and S.V. Panyukov, JETP Lett. **35**, 178 (1982) (Zh. Eksp.Teor. Fiz. **35**, 147 (1982)).
[3] V.V. Ryazanov, V.A. Oboznov, A.Yu. Rusanov, A.V. Veretennikov, A.A. Golubov, and J. Aarts, Phys. Rev. Lett. **86**, 2427 (2001); V.A. Oboznov, V.V. Bolginov, A.K. Feofanov, V.V. Ryazanov, and A.I. Buzdin, *ibid* **96**, 197003 (2006).
[4] T. Kontos, M. Aprili, J. Lesueur, F. Genet, B. Stephanidis, and R. Boursier, Phys. Rev. Lett. **89**, 137007 (2002).
[5] Y. Blum, A. Tsukernik, M. Karpovski, and A. Palevski, Phys. Rev. Lett. **89**, 187004 (2002).
[6] A. Bauer, J. Bentner, M. Aprili, M.L. Della-Rocca, M. Reinwald, W. Wegscheider, and C. Strunk, Phys. Rev. Lett. **92**, 217001 (2004).
[7] H. Sellier, C. Baraduc, F. Lefloch, R. Calemczuk, Phys. Rev. Lett. **92**, 257005 (2004).
[8] V. Shelukhin, A. Tsukernik, M. Karpovski, Y. Blum, K. B. Efetov, A.F. Volkov, T. Champel, M. Eschrig, T. Lofwander, G. Schon, A. Palevski, Phys. Rev. B **73**,174506 (2006).
[9] A. A. Bannykh, J. Pfeiffer, V. S. Stolyarov, I. E. Batov, V. V. Ryazanov, M. Weides, Phys. Rev. B **79**, 054501 (2009).
[10] F.S. Bergeret, A.F. Volkov, K.B. Efetov, Phys. Rev.Lett. **86**, 4096 (2001).
[11] R. S. Keizer, S. T. B. Goennenwein, T. M. Klapwijk, G. Miao, G. Xiao, A. Gupta, Nature **439**, 825 (2006).
[12] I. Sosnin, H. Cho, V. T. Petrashov, and A. F. Volkov, Phys. Rev. Lett. **96,** 157002 (2006).
[13] T. S. Khaire, M. A. Khasawneh, W. P. Pratt, Jr., N. O. Birge, Phys. Rev. Lett. **104**, 137002 (2010); Carolin Klose et al., *ibid* **108**, 127002 (2012).
[14] M. S. Anwar, F. Czeschka, M. Hesselberth, M. Porcu, J. Aarts, Phys. Rev. B **82**, 100501 (2010).
[15] D. Sprungmann, K. Westerholt, H. Zabel, M. Weides, H. Kohlstedt, Phys. Rev. B **82**, 060505 (2010).
[16] J. W. A. Robinson, J.D.S. Witt, M. G. Blamire, Science **329**, 59 (2010).
[17] J. Wang, M. Singh, M. Tian, N. Kumar, B. Liu, C. Shi, J. K. Jain, N. Samarth, T. E. Mallouk, and M. H. W. Chan, Nature Physics **6**, 389 (2010).
[18] A. Buzdin, Rev. Mod. Phys. **77**, 935 (2005).
[19] F.S. Bergeret, A. F. Volkov, K. B. Efetov, Rev. Mod. Phys. **77**, 1321 (2005).
[20] M. Eschrig, Physics Today, **64**, 43 (2011).
[21] A.A. Golubov, M.Y. Kupriyanov, and E.Il'ichev, Rev. Mod. Phys. **76,** 411 (2004).
[22] K. B. Efetov, I. A. Garifullin, A. F. Volkov and K. Westerholt, in *Magnetic Heterostructures. Advances and Perspectives in Spinstructures and Spintransport. Series Springer Tracts in Modern Physics, Vol. 227*, edited by H. Zabel and S. D. Bader (Springer, Berlin Heidelberg, 2007); in *Magnetic Nanostructures. Spin Dynamics and Spin Transport. Series Springer Tracts in Modern Physics, Vol. 246* (Springer, Berlin Heidelberg, 2012).
[23] X. Hao, J.S. Moodera, and R. Meservey, Phys. Rev. B **42**, 8235 (1990)
[24] T.S. Santos, J.S. Moodera, K.V. Raman, E. Negusse, J. Holroyd, J. Dvorak, M. Liberati, Y.U. Idzerda, and E. Arenholz, Phys. Rev. Lett. **101**, 147201 (2008).
[25] A.V. Zaitsev, JETP **59**, 1015 (1984) (Zh.Eksp.Teor. Fiz. **86**, 1742,(1984)) .
[26] A. Millis, D. Rainer, and J. A. Sauls, Phys. Rev. B **38**, 4504 (1988).
[27] Y. Tanaka and S. Kashiwaya, Physica C **274**, 357 (1997).
[28] M. Fogelström, Phys. Rev. B **62**, 11812 (2000).
[29] A. Cottet, D. Huertas-Hernando, W. Belzig and Yu. V. Nazarov, Phys. Rev. B **80**, 184511 (2009).
[30] M.S. Kalenkov, A. V. Galaktionov, and A. D. Zaikin, Phys. Rev. B **79**, 014521 (2009).
[31] S. Kawabata, Y. Asano, Y. Tanaka, A. A. Golubov, and S. Kashiwaya, Phys. Rev. Lett **104**, 117002 (2010)
[32] I. Zutic and O.T. Valls , Phys. Rev. B **61**, 1555 (2000).
[33] G.E. Blonder, M. Tinkham, and T.M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).
[34] J. Linder and A. Sudbo, Phys. Rev. **B** 75, 134509 (2007).
[35] J. Linder, T. Yokoyama, A. Sudbo, and M. Eschrig, Phys. Rev. Lett. **102**, 107008 (2009), J. Linder, T. Yokoyama, and A. Sudbø, Phys. Rev. **79**, 054523 (2009), J. Linder, A. Sudbo, T. Yokoyama, R. Grein and M. Eschrig, Phys. Rev. B **81**, 214504 (2010).
[36] F. S. Bergeret, A. Verso, and A. F. Volkov, Phys. Rev. B **86**, 060506(R) (2012)
[37] K. Senapati, M. G. Blamire, and Z. H. Barber, Nat. Mater. **10**, 1 (2011).
[38] M. Yu. Kuprianov, V.F. Lukichev, JETP **67**, 1163 (1988) (Zh. Eksp.Teor. Fiz. **94**, 139 (1988))
[39] J.R. Schrieffer, *Superconductivity* (Benjamin, New York,(1964)).
[40] I.O. Kulik and I.K. Yanson, *Josephson effects in superconducting tunnel structures*, Keter Press, Jerusalem (1972).
[41] A.Barone and G.Paterno, *Physics and applications of the Josephson effect*, Wiley, NY (1982).
[42] E. L. Wolf, *Principles of electron tunneling spectroscopyxford,* (University Press, Oxford,1985)
[43] A. F. Volkov, JETP **41**, 376 (1975) (Zh. Eksp.Teor. Fiz. 68, 756 (1975)),
[44] S. N. Artemenko and A. F. Volkov, JETP **60**, 395 (1984) (Zh. Eksp.Teor. Fiz. **87**, 691 (1984)),
[45] A. Moor, A. F. Volkov, K. B. Efetov, Phys. Rev. B **85**, 014523 (2012).
[46] F. S. Bergeret, A. Levy Yeyati, and A. Martín-Rodero, Phys. Rev. B **72**, 064524 (2005).
[47] F.S. Bergeret, A.F. Volkov, K.B. Efetov, Phys. Rev.Lett. **86**, 3140(2001).

[48] T. Tokuyasu, J. A. Sauls, and D. Rainer, Phys. Rev. B **38**, 8823 (1988).
[49] We assume, as usual, that the exchange field vector is parallel to the magnetization one.
[50] C. W. J. Beenakker, Phys. Rev. B **46**, 12841 (1992).
[51] A. F. Volkov, A. V. Zaitsev, and T. M. Klapwijk, Physica C **210** 21 (1993).
[52] A. Ozaeta, A. S. Vasenko, F. W. J. Hekking, and F. S. Bergeret, Phys. Rev. B **86**, 060509 (2012).
[53] A. Kastalsky, A. W. Kleinsasser, L. H. Greene, R. Bhat, F. P. Milliken, and J. P. Harbison, Phys. Rev. Lett. **67**, 3026 (1991).
[54] D. Quirion, C. Hoffmann, F. Lefloch, and M. Sanquer, Phys. Rev. B **65**, 100508(R) (2002).
[55] C. J. Lambert, R. Raimondi, V. Sweeney, and A. F. Volkov, Phys. Rev. B **55**, 6015 (1997).
[56] A. L. Shelankov, Sov. Phys. Solid State **26**, 981 (1984)
[57] M. Eschrig, Phys. Rev. B **61**, 9061 (2000); *ibid* **80**, 134511 (2009).
[58] A. F. Volkov, JETP Lett. **55**, 746 (1992) (Zh. Eksp.Teor. Fiz. 55, 713 (1992)); Phys. Lett. A **174**, 144 (1993).
[59] B. J. van Wees, P. de Vries, P. Magnée, and T. M. Klapwijk, Phys. Rev. Lett. **69**, 510 (1992).
[60] F. W. J. Hekking and Y. V. Nazarov, Phys. Rev. B **49**, 6847 (1994).
[61] A. V. Burmistrova and I. A. Devyatov, JETP Letters **96**, 391 (2012) (Zh. Eksp.Teor. Fiz **96**, 430 (2012)).
[62] A. V. Burmistrova, I. A. Devyatov, Alexander A. Golubov, Keiji Yada, Yukio Tanaka, arXiv:1210.1479
[63] M. Julliere, Phys. Lett. **54A**. 225 (1975).
[64] J. C. Slonczewski, Phys. Rev. B **39**, 6995 (1989).
[65] Note, that Kupriyanov-Lukichev BCs, although quite useful, are phenomenological ones. The authors of Ref.[38] started with the Zaitsev's microscopic BCs[25], but finally their derivation of the BCs in the dirty limit was not rigorous. For a discussion of this subject see Ref.[55]
[66] To be more exact in the experiments of Refs.[53,54] a highly doped semiconductor-Shottky barrier-superconductor structure was explored